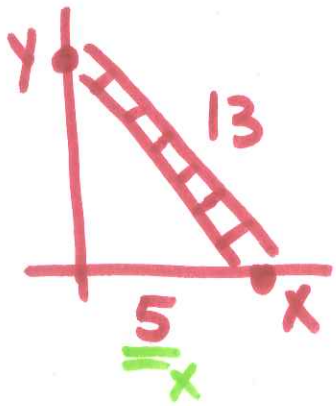
Chapter 7A - Day 2

Ex: a 13 ft ladder rests against a wall. If the bottom slides away from the wall at a rate of 1 ft/sec ($\frac{dx}{dt}$), how fast is the top of the ladder sliding down the wall when the bottom of the ladder is 5 ft from the wall? ($\frac{dy}{dt}$)



$$x^2 + y^2 = h^2$$

solve for y

$$5^2 + y^2 = 13^2$$

$$y^2 = 169 - 25 = 144$$

$$y = \sqrt{144} = 12_y$$

differentiate $x^2 + y^2 = 13^2$ with respect to t

$$2x \cdot \frac{dx}{dt} + 2y \cdot \frac{dy}{dt} = 0$$

$$2(5) \cdot (1) + 2(12) \cdot \frac{dy}{dt} = 0$$

$$24\frac{dy}{dt} = -10 \rightarrow \frac{dy}{dt} = \frac{-10}{24} \approx -.41\overline{6} \text{ ft/sec}$$

*Note: the answer is negative since the ladder is falling down and distance y is decreasing.

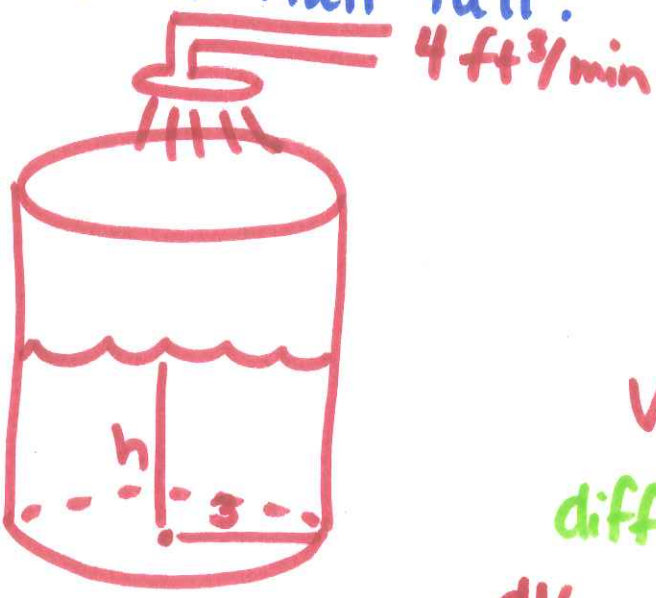
The top of the ladder is falling down the wall at

$\frac{10}{24}$ or $.41\overline{6}$ ft/sec

Ex: A cylindrical water tank is being filled at the rate of 4 ft³/min. The radius of the tank is 3 ft. How fast is the level of the water in the tank rising when the tank is half full?

dV/dt

dh/dt



Volume of a Cylinder

$V = \pi r^2 h$

Since $r = 3$

$V = \pi(3^2)h = 9\pi h$

differentiate $V = 9\pi h$

$$\frac{dV}{dt} = 9\pi \cdot \frac{dh}{dt}$$

$$4 = 9\pi \frac{dh}{dt}$$

$$\frac{dh}{dt} = \frac{4}{9\pi} \approx .14147 \text{ ft/min}$$

Ex: An annual advertising revenue for a newspaper is $R(x) = .4x^2 + 6x + 150$ thousand dollars when x is in thousands. Current circulation is 8,000 papers and is increasing by 1,000 papers per year. Two years from now, at what rate will the advertising revenue be increasing?

$dx/dt = 1$

dR/dt

differentiate $R(x) = .4x^2 + 6x + 150$

$$\frac{dR}{dt} = (.8x + 6) \cdot \frac{dx}{dt}$$

in 2 years $x = 8000 + 1000(2) = 10{,}000$ so $x = 10$

Plug in what we know

$$\frac{dR}{dt} = (.8(10) + 6) \cdot (1)$$

$$= 8 + 6 = 14$$

$14,000 per year

Ex: A stock is increasing in value by $8 per share per year. ($8 → dP/dt)

An investor buys shares at a rate of 20 shares per year. (20 shares per year → dn/dt)

How fast is the value of his stock growing when the stock price is $40 per share and the investor owns 150 shares? (How fast → dV/dt; $40 per share → P; 150 shares → n)

What is the total value?

n = # of shares
P = $ per share
V = total value

$V = np$

take derivative!

$$\frac{dV}{dt} = \frac{dn}{dt} \cdot P + n \cdot \frac{dp}{dt}$$

$$\frac{dV}{dt} = (20)\cdot(40) + (150)\cdot(8)$$

$$= 800 + 1200$$

$$= \$2{,}000 \text{ per year}$$